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Propagation Features of Beams with Axially Symmetric Polarization

A.V. Nesterov, V.G. Niziev
Institute on Laser & Information Technologies of RAS
Shatura, Moscow region, 140700 RUSSIA

Abstract. The general solution of the wave equation for the axially symmetric polarized (ASP) beams consists of two independent solutions: azimuthally polarized beam and beam with longitudinal and radial field components. Maximum of the longitudinal field is at the beam axis where the transverse component equals to zero. In spite of the longitudinal component is maximum in the waist it doesn't contribute to beam divergence here, therefore the wave front of ASP-beams is flat in the focal plane. The ASP-beams are free from polarization aberrations, which are inherent for linearly polarized beams passing through a lens with large annular apertures, and these beams are prospective for experiments on obtaining "diffraction free" beams. The formulae and their analysis for electromagnetic field in the case of sharp focusing of ASP-beams in Debye approximation are presented.

1. Introduction

Beams with axially symmetric polarization (ASP) are of special interest as from the theoretical viewpoint and from the viewpoint of their applications. The full axial symmetry of these beams (including amplitude, phase and polarization) makes them optimum in many applications: light transmission by metallic waveguides, laser materials processing, experiments on inertial fusion and others [1-4]. An effective method for generating such beams by using a polarization selective intra-cavity mirror was demonstrated in paper [5]. According to this, the task of describing the properties of axially symmetric polarized beams such as propagation, diffraction and focusing becomes actual. The main feature of ASP-beams is that the angle between the electric vector at a point and the radius directed to this point is constant over the beam cross-section. Well known examples are radially and azimuthally polarized beams. The authors of paper [6] showed that the symmetry of the intensity distribution of ASP-beams is a direct consequence of their polarization symmetry.

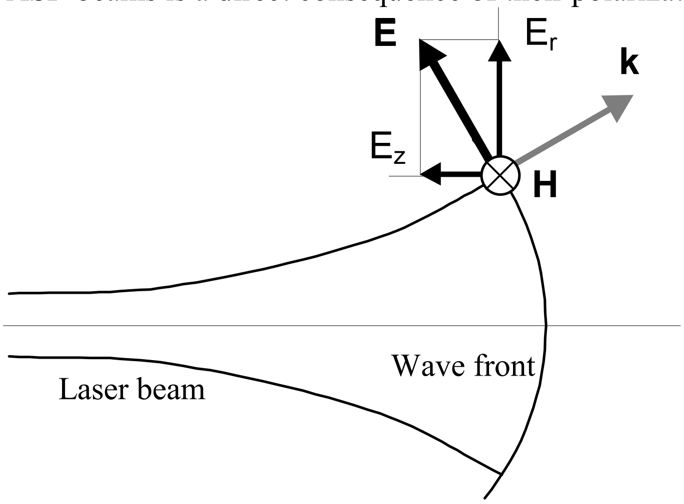
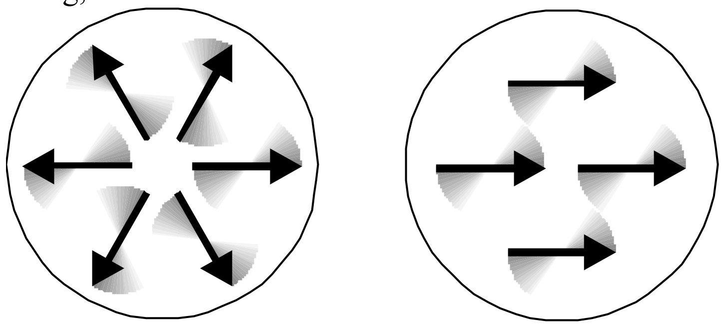


Fig. 1. Schematic view of relative location of the electromagnetic field and the wave vector **k** for a beam with r-z directed field.

The usual scalar wave equation is derived from the vector wave equation in assumption of homogeneous polarization distribution over the beam cross-section. In this case one neglects a longitudinal component of the electric field naturally connected with the beam divergence in such approach. In the case of ASP-beams the maximum amplitude is on the axis where the transversal

component (**E** or **H**) of the field equals to zero. Therefore there is no reason to neglect the longitudinal field here.
The aim of the paper is analytical investigating features of propagation of ASP-beams for creating comparatively full and logical description of such beams in different appearances: paraxial propagation, sharp focusing, diffraction.



(a) (b)

Fig. 2. The helical (a) and conventional (b) modes with circular polarization. Arrows show instant direction of electric vector and distribution of its phase over the cross-section. The tail area of the arrows shows direction of rotation.

2. General approach to ASP-beams

The equation for "slowly varying amplitude of monochromatic axially symmetric polarized field (without multiplier exp(ikz)) has the form

$$\left(\hat{L}_r + \hat{L}_\varphi + \hat{L}_z\right)\mathbf{E} = 0, \tag{1}$$

$$\hat{L}_r = \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r}, \qquad \hat{L}_\varphi = \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2}, \qquad \hat{L}_z = 2ik\frac{\partial}{\partial z} + \frac{\partial^2}{\partial z^2},$$

$$\mathbf{E} = E_r \cdot \mathbf{n}_r + E_\varphi \cdot \mathbf{n}_\varphi + E_z \cdot \mathbf{n}_z.$$

The field components E_r, E_φ, E_z are independent on φ and are functions of r and z. Taking into account the expressions $\partial n_r / \partial \varphi = -n_\varphi$, $\partial n_\varphi / \partial \varphi = n_r$ (the other derivatives for the unit vectors equal to zero) we get from (1) the following equations for the field components

$$\left(\hat{L}_r - \frac{1}{r^2} + \hat{L}_z\right)E_r = 0, \tag{2a}$$

$$\left(\hat{L}_r - \frac{1}{r^2} + \hat{L}_z\right)E_\varphi = 0, \tag{2b}$$

$$\left(\hat{L}_r + \hat{L}_z\right)E_z = 0. \tag{2c}$$

Maxwell equation in the form

$$\frac{1}{r}\frac{\partial (rE_r)}{\partial r} + \frac{\partial E_z}{\partial z} = 0 \tag{3}$$

has to add to the system (2). Thus, the general solution for an ASP-beam consists of two independent solutions for an azimuthally polarized beam (2b) and for a polarized beam with radial and longitudinal components of field:

$$\mathbf{E} = k_1 \cdot E_\varphi \cdot \mathbf{n}_\varphi + k_2 \cdot (E_r \cdot \mathbf{n}_r + E_z \cdot \mathbf{n}_z),$$

where k_1 and k_2 are arbitrary complex coefficients. Figure 1 illustrates a beam with r-z electric field components (k_1=0). The beams with azimuthal, radial or intermediate direction of polarization are obtained by zero phase shift $\Delta\varphi$ between two solutions (2, 3) and different magnitudes of k_1 and k_2. If $|k_1|=|k_2|$ and $\Delta\varphi=\pi/2$, the beam is circularly polarized and the phase of electrical vector equals to the azimuthal angle. It is helical mode [7] with circular polarization (Figure 2a). It is clear that this beam penetrating through the $\lambda/4$ retarder becomes a classical linearly polarized mode. In the contrary to the helical mode with circular polarization the radially polarized beam with "normal" phase distribution over the cross section is shown on Figure 2b.

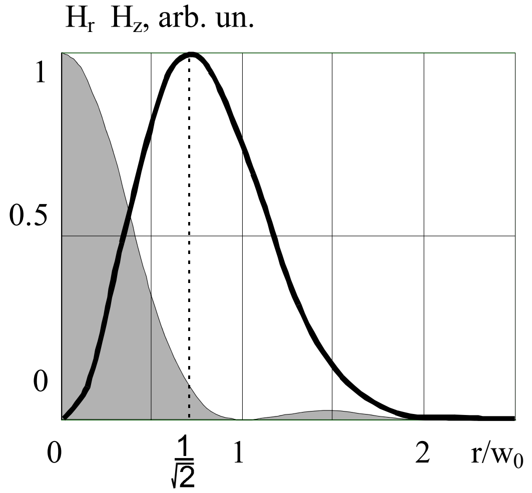


Fig. 3. Radial distribution of longitudinal components H_z (filled region) and radial H_r (curve), obtained from (5).

The physical reason of existence of two independent solutions is that the azimuthally polarized beam retains its state of polarization but the ratio of longitudinal and transverse components of field changes.

The authors of [8] considered in details the famous paradox of classic paraxial approximation in the study of modes in spherical laser resonators. The solution for plane polarized mode has spherical wavefront. This zero order approximation contradicts to exact Maxwell equations. The first-order field is found to be a longitudinal field.

In the contrary to plane polarization in our case for ASP beams we can contend that:

a) The solutions with or without longitudinal component of field are separated each other.
b) It is true for common case without paraxial restriction.
c) Both solutions can be found by the same methods. We should start calculation from the azimuthally directed field E or H, and then calculate radial or longitudinal components of H or E respectively.

3. Self-similar solutions for ASP-beams in paraxial approximation

The self-similar solutions of the equation (2b) in paraxial approximation

$$\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial E_\varphi}{\partial r}-\frac{1}{r^2}E_\varphi+2ik\frac{\partial E_\varphi}{\partial z}=0 \tag{4}$$

are well known Laguerre-Gauss beams TEM_{p1*}. The components of electric and magnetic field of the beam TEM_{01*} can be calculated:

$$E_\varphi=\frac{2}{\sqrt{\pi}}\cdot\frac{1}{w}\cdot R\cdot\exp\left(-R^2\right)\cdot\exp(i\theta),$$

$$H_r = \frac{i}{\pi\sqrt{\pi}}\frac{\lambda}{w}\frac{1}{z_0} R\cdot(m+in)\cdot\exp\left(-R^2\right)\cdot\exp(i\theta), \tag{5}$$

$$H_z = -2\frac{i}{\pi\sqrt{\pi}}\cdot\frac{\lambda}{w}\frac{1}{w}\cdot\left(1-R^2-iZR^2\right)\cdot\exp\left(-R^2\right)\cdot\exp(i\theta),$$

where R = r/w, $w^2 = w_0^2\cdot\left(1+Z^2\right)$, Z = z/z$_o$, $z_0 = \dfrac{\pi w_0^2}{\lambda}$, w_o is the beam radius in the waist, λ is the wavelength;

$$\theta = 2\mathrm{arctg}Z - 2Z\frac{z_0^2}{w_0^2} - ZR^2;\quad m = 2Z\frac{w_0^2}{w^2}\left(R^2-1\right);\quad n = \frac{2w_0^2}{w^2} - 2\frac{z_0^2}{w_0^2} + R^2\frac{Z^2-1}{Z^2+1}.$$

One can calculate the amplitude of magnetic vector and its direction. Note that H_z has the phase shift $\pi/2$ relative to E_φ in cross-section Z=0. Therefore the time averaged wave vector has no radial component over this cross-section and the wave front is flat. Figure 3 shows the vector **H** distribution in the waist. Maximum values of magnetic vector H_z and H_r is achieved at R_0=0 and $R_0=1/\sqrt{2}$ respectively. Their ratio is of order of λ/w_0:

$$\frac{(H_z)_{max}}{(H_r)_{max}} = \frac{(H_z)_{R_0=0}}{(H_r)_{R_0=1/\sqrt{2}}} \sim \frac{\lambda}{w_0} \tag{6}$$

It is evident that these results are valid by mutual substitute of **E** and **H**. The self-similar solutions (4) can be also resonator modes.

4. Huygens-Fresnel integral for ASP-beams

We consider the ASP-beams with axially symmetric intensity distribution. The solution of equation (4) in general case is obtained by the method of separation of variables and can be expressed by the integral:

$$E_\varphi(r,z) = \int_0^\infty A(\sigma^2)\cdot J_1(\sigma r)\cdot\exp\left(-\frac{i}{2k}\sigma^2 z\right)\cdot d\sigma^2\ . \tag{7}$$

Here σ^2 is the constant of separation.
We consider δ-ring source at z=0:

$$\delta(r-r_0) = \int_0^\infty A(\sigma^2)\cdot J_1(\sigma r)\cdot d\sigma^2$$

Using the known formula

$$\delta(r-r_0) = r_0\int_0^\infty J_1(vr)\cdot J_1(vr_0)\cdot v\cdot dv\ .$$

we find $A(\sigma^2)$ and obtain from (7) the expression

$$G_\varphi(r_0,r,z) = \frac{r_0}{2}\int_0^\infty J_1(\sigma r_0)\cdot J_1(\sigma r)\cdot\exp\left(-\frac{i}{2k}\sigma^2 z\right)\cdot d\sigma^2\ ,$$

After integrating according to

$$\int_0^\infty \exp(-iax)\cdot J_1(2\gamma\sqrt{x})\cdot J_1(2\delta\sqrt{x})\cdot dx = \frac{1}{ia}\cdot J_1\left(\frac{2\gamma\delta}{a}\right)\cdot\exp\left(-\frac{\gamma^2+\delta^2}{ia}\right),$$

the Green function acquires the form:

$$G_\varphi(r_0,r,z) = \frac{kr_0}{iz}J_1\left(\frac{krr_0}{z}\right)\cdot\exp\left(ik\left(z+\frac{r^2+r_0^2}{2z}\right)\right). \tag{8}$$

Equation (1) describes the slow amplitude without the factor exp(ikz), as introduced in (8). Thus, the distribution of azimuthally polarized beam in paraxial approximation is given by the integral

$$E_{\varphi}(r', z') = \int_0^{\infty} E^0_{\varphi}(r_0) \cdot G_{\varphi}(r_0, r', z') \cdot dr_0 \, . \tag{9}$$

Another more complicated method of obtaining expression (8) based on vector superposition of two scalar solutions of wave equations for two transverse coordinates, was used in paper [9].
In the case of r-z directed field the electric vector in cross-section z=0 is given by:

$$\mathbf{E}^0(r_0, z=0) = E^0_r(r_0, z=0) \cdot \mathbf{n}_r + E^0_z(r_0, z=0) \cdot \mathbf{n}_z \, .$$

Calculating the magnetic component from

$$H^0_{\varphi}(r, z=0) = -\frac{i}{k} \cdot \left(\frac{\partial E^0_r}{\partial z} - \frac{\partial E^0_z}{\partial r} \right)$$

and applying formulae (8) and (9) for azimuthally directed field we obtain the solution for the vector **H** and therefore for **E**.

5. Fraunhofer diffraction on a ring slit

One can solve the task of Fraunhofer diffraction of azimuthally polarized beams on a ring slit on the base of the Green function (8). By analogy with [10], the intensity distribution in the far field expressed by vectorial angle θ is given by

$$\frac{dI}{I_0} = \frac{k^2 \cdot R \cdot \Delta R}{2 \cdot \pi} \cdot J_1^2(kR\theta) \cdot d\Omega \, . \tag{10}$$

Here R and ΔR are the radius and the width of the radiating ring in the cross section z=0, dΩ is the element of solid angle.
For comparison, the formula for diffraction of a linearly polarized beam on a ring slit in the same notation has the form

$$\frac{dI}{I_0} = \frac{k^2 \cdot R \cdot \Delta R}{2 \cdot \pi} \cdot J_0^2(kR\theta) \cdot d\Omega \tag{11}$$

The “diffraction free” beams were reported to be obtained experimentally [11]. A linearly polarized beam passed the ring slit and then the lens positioned at the focal distance from the slit in that experiment. After passing the lens the beam acquired a Bessel field distribution, cut by lens aperture. Degree of approximation to the ideal Bessel function in this case is dependent on lens aperture. But the larger are lens aperture, the larger are polarization aberrations. Using ASP-beams in this scheme one can remove this disadvantage and provide “diffraction free” beam with intensity distribution in the form of first order Bessel function.
As for the same diffraction of the beam with r-z direction of field, the intensity distribution of the radial component has also the form (10) and the longitudinal component is described by the expression (11) with the additional factor $1/(k \cdot R)^2$.

6. Debye approximation for sharp focusing of ASP-beams

The task of sharp focusing is outside the above discussed paraxial approximation. It is usually solved with the Debye method. According to this method, the field distribution in the focal region is formed by wave rays, converging inside the cone restricted by the aperture of the optical system. The phase of the wave ray is described by multiplier exp(i**kr**),

$$\mathbf{k} = (-k \cdot \sin\theta \cdot \cos\beta, \ -k \cdot \sin\theta \cdot \sin\beta, -k \cdot \cos\alpha),$$

$$\mathbf{r} = (r \cdot \sin\theta \cdot \cos\varphi, \ r \cdot \sin\theta \cdot \sin\varphi, \ r \cdot \cos\theta).$$

where **k** is the ray vector and **r** is radius-vector in coordinate system connected to the focus. The polarization direction is perpendicular to **k** and is determined relative to the angle between **k** and the optical lens axis.

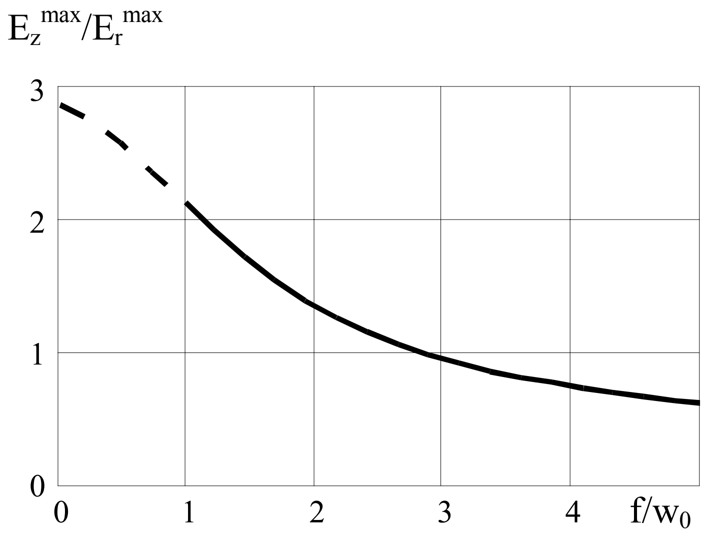


Fig. 4. The ratio of maximum amplitudes of longitudinal and transverse electrical fields in the focal plane versus the focal length in Debye approximation.

According to [12,13], there is the following expression for the electric vector in the general case:

$$\mathbf{E} = \frac{-ik}{2\pi}\int_0^{2\pi}\int_0^{\theta_1} A(\alpha)\cdot\mathbf{p}(\alpha,\beta)\cdot\exp(i\mathbf{kr})\cdot\sin\alpha\cdot d\alpha d\beta\,.$$

Here θ_1 is the angle of semiaperture of the optical system, $\mathbf{p}(\alpha,\beta)$ is the unit vector which coincides with the direction of **E** in the region extending from the lens to the focal plane, $A(\alpha)$ is the coefficient connected with the beam distribution $A_0(\rho)$ incident upon the focusing system and its focal length f. $A(\alpha)=A_0(f\sin\alpha)\cdot\sqrt{\cos\alpha}$ for aplanatic system.

The unit vector $\mathbf{p}(\alpha,$□$\beta)$ in the case of ASP-beams can be expressed in the form $e^{i\beta}$ for azimuthal and $e^{i(\beta+\pi/2)}$ for r-z types of polarization. Taking into account this representation, the expressions for vector **E** in the focal region are given by:

$$E_r=0, \qquad E_z=0$$

$$E_\varphi(u,v) = k\int_0^{\theta_1} A(\alpha)\cdot J_1(v\cdot\sin\alpha/\sin\theta_1)\cdot\exp(i\cdot u\cdot\cos\alpha/\sin^2\theta_1)\sin\alpha d\alpha d\beta$$

for azimuthal polarization, and

$$E_r(u,v) = \frac{k}{2}\int_0^{\theta_1} A(\alpha)\cdot J_1(v\cdot\sin\alpha/\sin\theta_1)\cdot\exp(i\cdot u\cdot\cos\alpha/\sin^2\theta_1)\sin 2\alpha d\alpha d\beta$$

$$E_z(u,v) = -i\cdot k\int_0^{\theta_1} A(\alpha)\cdot J_0(v\cdot\sin\alpha/\sin\theta_1)\cdot\exp(i\cdot u\cdot\cos\alpha/\sin^2\theta_1)\sin^2\alpha d\alpha d\beta$$

$$E_\varphi=0$$

for radial polarization, where v and u are the optical coordinates:

$$v = k\cdot r\cdot \sin\theta\cdot\sin\theta_1 = k\cdot\rho\cdot\sin\theta_1, \qquad u = k\cdot r\cdot\cos\theta\cdot\sin^2\theta_1 = k\cdot z\cdot\sigma\iota\nu^2\theta_1.$$

We used for derivation of these expressions the integral:

$$\int_0^{2\pi} \exp[i\cdot n\cdot\beta + i\cdot t\cdot\cos(\beta-\gamma)]\cdot d\beta = 2\pi\cdot i^n\cdot J_n(t)\exp(i\cdot n\cdot\gamma)$$

The field with electric vector parallel to the optical lens axis with maximum on this axis appears in the case of radially polarized beam.

Figure 4 illustrates the ratio of maximum amplitudes of longitudinal and transverse electrical fields in the focal plane versus the focal length in Debye approximation. The part of the longitudinal components decreases at increasing of focal length. The longitudinal component possesses the larger divergence in comparison with the transverse component (Figure 5).

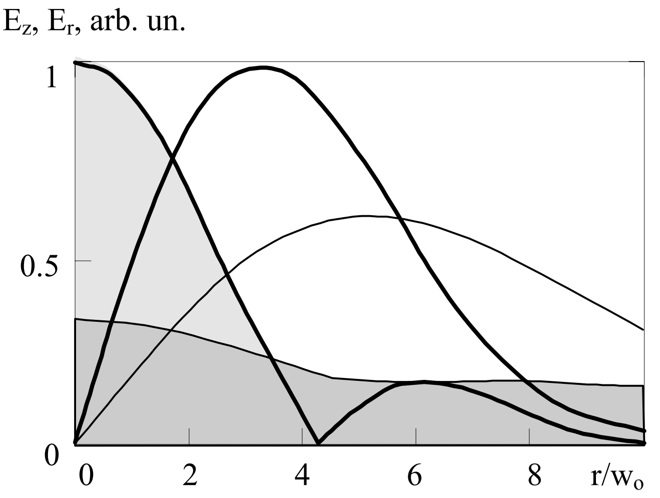


Fig. 5. The illustration of relative distribution of E_z (filled regions) and E_r (curves) at the waist z=0 (thick lines) and at the distance $z=8w_0$ (thin lines). The focal distance is $f=2.6w_0$. The scale of all curves is the same.

7. Green function for non-paraxial ASP-beams.

The vector wave equation (1) is transformed to the scalar equation for the azimuthally polarized beam:

$$\frac{1}{r}\frac{\partial}{\partial r} r\frac{\partial u_\varphi}{\partial r} - \frac{1}{r^2}u_\varphi + \frac{\partial^2 u_\varphi}{\partial z^2} + 2ik\frac{\partial u_\varphi}{\partial z} = 0\,.$$

The Green function of this equation can be calculated the following way:

$$G(r_0, r, z) = \frac{r_0}{2}\int_0^\infty J_1(\sigma r_0)\cdot J_1(\sigma r)\cdot\exp\left[\left(-ik\pm\sqrt{\sigma^2-k^2}\right)\cdot z\right]\cdot d\sigma^2\,.$$

This formula for non-paraxial beams is a generalisation of the expression (8).

8. Conclusion

The general solution for the ASP-beams consists of two independent solutions: an azimuthally polarized beam and a beam with longitudinal and radial components of field. Maximum of the longitudinal field is on the beam axis where the transverse component equals to zero. The ratio of maximum amplitudes of longitudinal and transverse components is of order of

λ/w_0 (λ is the wavelength, w_0 is waist radius). The longitudinal component does not contribute to beam divergence in the waist, therefore the wave front of ASP-beams is flat in the focal plane.

Fraunhofer diffraction of ASP-beams on a ring slit gives a field distribution proportional to the first order Bessel function. The ASP-beams are free from polarization aberrations, which are inherent for linearly polarized beams passing through a lens with large annual aperture, and are prospective for experiments on obtaining “diffraction free” beams.

The formulae and their analysis for electromagnetic field in the case of sharp focusing of ASP-beams are presented.

References

1. Balmer J.E., Donaldson T.P.
   Resonance Absorption of 1.06 μm laser radiation in laser-generated plasma.
   Phys. Rev. Lett., 1977, v39, n17, p1084-1087.
2. Romea R.D. and Kimura W.D. Modeling of inverse Cherenkov laser acceleration with axicon laser beam focusing. Physical Review D, 1990, v.42, n.5, p.1807.
3. Yariv A. Propagation of optical beams in fibers. Optical Electronics 3rd ed., CBS College Publishing, New York, N.Y., 67-66, 1985.
4. Niziev V.G., Nesterov A.V.1999 Influence of Beam Polarization on Laser Cutting Efficiency Journal of Physics D Appl. Phys. V.32, p. 1455-1461
5. Nesterov A.V., Niziev V.G., Yakunin V.P. 1999 Generation of high-power radially polarized beam Journal of Physics D Appl. Phys. V.32, p. 2871-2875
6. Nesterov A.V., Niziev V.G. 2000 Laser beams with axially symmetric polarization Journal of Physics D Appl. Phys. V.33, p.1324-1329.
7. Arsenyan T.I., Fedotov N.N., Kornienko L.S., Korolenko P.V., Kulyagina E.A., Petrova G.V. Laser beam with helical wavefront dislocations and their applications in the diagnostical and metrological systems. SPIE v.2713, p.453, 1995.
8. M. Lax, W.H. Louisell, W.B. Mc Knight From Maxwell to paraxial wave optics.
   Physical Review A 1975, v. 11, n. 4, p. 1365-1370.
9. R. Jordan, D. Hall. Free-space azimuthal paraxial equation: the azimuthal Bessel-Gauss beam solution Opt. Lett., 1994, v.19, N7, p.427-429.
10. L.D.Landau and E.M. Lifshitz, Classical Theory of Fields Pergamon Press, Ozford, 1982.
11. Durnin J., Miceli J.J., Eberly J.H. Diffraction-Free Beams
    Physical Review Letters 1987, v.58, 15, p.1499-1501.
12. Stamnes J.J. Waves in Focal Regions The Adam Hilger Series on Optics and Optoelectronics. IOP Publishing Ltd. Adam Hilder, Bristol and Boston 1986.
13. Solimeno S., Crosignani B., DiPorto P.
    Guiding, Diffraction and Confinement of Optical Radiation
    New York: Academic Press, 1986.